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DISCUSSION OF MERIT CONTESTS IN COLLEGE EXAMINATIONS BY THE METHOD OF LEAST SQUARES.

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[A paper read before the Philosophical Society of Washington.]

It is well known that it is customary in schools and colleges to estimate merit on the basis of 100 being perfect. If then a number of students 1, 2, 3, ...n receive from judges A, B, C, ...L (number$=m$) the estimates of merit a_1, b_1, c_1, ... l_1 ; a_2, b_2, c_2, ...l_2 ; ...... respectively, it is required to find from these discrepant data the most probable merit of each student as well as the personal error of each judge.

Take the case in which three judges A, B, C, have given estimates of merit to seven students, and let the estimates for the first student be:

$a_1=87$; $b_1=70$; $c_1=70$.

Let $M_1=$true or most probable merit.

$\Delta a_1=$error of judge A,

$\Delta b_1=$error of judge B,

$\Delta c_1=$error of judge C,

then we have undoubtedly,

$$M_1=\tfrac{1}{3}(a_1+b_1+c_1)\pm 0.6745\sqrt{\frac{\Delta a_1^2+\Delta b_1^2+\Delta c_1^2}{3\times 2}}$$

$$=75.7\pm 3.82 \text{ with weight } p_1=1.56.$$

If there was but this one student, then this would be the final answer to the question and the personal errors of the judges must be taken,

$\Delta a_1 = -11.3$,

$\Delta b_1 = +5.7$,

$\Delta c_1 = +5.7$.

If these same judges give estimates of merit to more than one student, we shall have more or less discrepant values of the errors of the judges from which a mean personal error may be determined. Now we have the following individual results for the seven students in the example,

$M_1 = 87 - 11.3 = 70 + 5.7 = 70 + 5.7 = 75.7 \pm 3.82$; $p_1 = 1.56$,

$M_2 = 92 - 4.3 = 76 + 11.7 = 95 - 7.3 = 87.7 \pm 3.98$; $p_2 = 1.44$,

$M_3 = 80 - 11.7 = 60 + 8.3 = 65 + 3.3 = 68.3 \pm 4.05$; $p_3 = 1.38$,

$M_4 = 93 - 8.3 = 68 + 16.7 = 93 - 8.3 = 84.7 \pm 5.62$; $p_4 = 0.71$,

$M_5 = 85 - 11.7 = 67 + 6.3 = 68 + 5.3 = 73.3 \pm 3.94$; $p_5 = 1.47$,

$M_6 = 95 - 7.3 = 78 + 9.7 = 90 - 2.3 = 87.7 \pm 3.40$; $p_6 = 1.98$,

$M_7 = 96 - 9.0 = 80 + 7.0 = 85 + 2.0 = 87.0 \pm 3.19$; $p_7 = 2.24$.

In examining these results we notice that judge *A* always over-estimates, judge *B* under-estimates, and that judge *C* is the least consistent of the three judges, as can be roughly seen from their ranges, being 7.4 for *A*, 11.0 for *B*, and 14.0 for *C*. To determine the personal errors of the judges, the arithmetical mean of their errors might be taken, but it is more rigorous to take their weighted mean, using the above weights, which are reciprocally proportional to the squares of the probable errors. We have thus,

$$\Delta a = \frac{[p\Delta a]}{[p]} \pm 0.6745 \sqrt{\frac{[p\Delta a^2] - [p]\Delta a^2}{[p](n-1)}}$$

=personal error of judge *A*, and similarly for the other judges. We then have the numerical results :

$\Delta a = -9.0 \pm 0.70$; $P_a = 1.53$.

$\Delta b = +8.7 \pm 0.79$; $P_b = 1.20$.

$\Delta c = +0.4 \pm 1.31$; $P_c = 0.44$.

It is obvious that if we correct the original estimates by these quantities, the corrected merits by judge *A* will be nearest the truth, those of *B* will be next best, and those of *C* will hardly be improved ; hence the merits must be redetermined by the formula,

$$M_r = a_r + \Delta a + \Delta^2 a_r = b_r + \Delta b + \Delta^2 b_r = c_r + \Delta c + \Delta^2 c_r,$$

$$= \frac{P_a(a_r + \Delta a) + P_b(b_r + \Delta b) + P_c(c_r + \Delta c)}{P_a + P_b + P_c}$$

$$\pm 0.6745 \sqrt{\frac{P_a(\Delta a^2 + \Delta^2 a_r^2) + P_b(\Delta b^2 + \Delta^2 b_r^2) + P_c(\Delta c^2 + \Delta^2 c_r^2)}{(P_a + P_b + P_c)(3-1)}}$$

which gives the following numerical results :

$M_1 = 78.0 - 0.8 = 78.7 - 1.5 = 70.4 + 6.8 = 77.2 \pm 4.14,$

$M_2 = 83.0 + 2.4 = 84.7 + 0.7 = 95.4 - 10.0 = 85.4 \pm 4.33,$

$M_3 = 71.0 - 1.7 = 68.7 + 0.6 = 65.4 + 3.9 = 69.3 \pm 4.03,$

$M_4 = 84.0 - 1.5 = 76.7 + 5.8 = 93.4 - 10.9 = 82.5 \pm 4.72,$

$M_5 = 76.0 - 1.2 = 75.7 - 0.9 = 68.4 + 6.4 = 74.8 \pm 4.12,$

$M_6 = 86.0 + 0.9 = 86.7 + 0.2 = 90.4 - 3.5 = 86.9 \pm 3.98,$

$M_7 = 87.0 + 0.4 = 88.7 - 1.3 = 85.4 + 2.0 = 87.4 \pm 3.97.$

These results show, as they should, that the corrected estimates of judge *A* came nearest to the correct value, *B's* next best, and *C's* hardly improved ; their ranges being 4.1, 7.3 and 17.7 respectively. We also notice that now contestant 7 has the highest merit while in the first approximation 2 and 6 came out best with a tie. The reason for this is that 2 and 6 received very high marks from judge *C*, which have very small weight in the second approximation. There is an apparent paradox in this, that the best values of the merits have nevertheless larger probable errors than those of the first approximation. Now each of the arithmetic means of the first approximation involves only three errors out of the 27. By correcting the estimates by the personal errors of the judges, a secondary effect of the remaining 24 errors is added in each case.

The second approximation, although sufficiently close to the true values may however, yet be improved. For we now have more correct actual errors of the judges as follows :

$\Delta a_1 = -9.8$; $\Delta b_1 = +7.2$; $\Delta c_1 = +7.2,$

$\Delta a_2 = -6.6$; $\Delta b_2 = +9.4$; $\Delta c_2 = -9.6,$

$\Delta a_3 = -10.7$; $\Delta b_3 = +9.3$; $\Delta c_3 = +4.3,$

$\Delta a_4 = -10.5$; $\Delta b_4 = +14.5$; $\Delta c_4 = -10.5,$

$\Delta a_5 = -10.2$; $\Delta b_5 = +7.8$; $\Delta c_5 = +6.8,$

$\Delta a_6 = -8.1$; $\Delta b_6 = +8.9$; $\Delta c_6 = -3.1,$

$\Delta a_7 = -8.6$; $\Delta b_7 = +7.4$; $\Delta c_7 = +2.4.$

The weighted means of these, weights being given according to their probable errors, will be new values of the personal errors Δa, Δb, Δc of the judges and applying these we obtain new values for the merits. It is easily seen however that this can only affect the 0.01 and though theoretically speaking an infinite number of approximations is required to obtain the most probable values of the merits we may safely regard the second approximation as sufficient.

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ON THE CIRCULAR POINTS AT INFINITY.

By E. D. ROE, JR., Associate Professor of Mathematics in Oberlin College, Oberlin, Ohio.

I. THE COORDINATE SYSTEM. In the following discussion, in addition to the usual Cartesion coördinates, homogeneous point and line coördinates will be used. They are related to Cartesian coördinates as follows :*

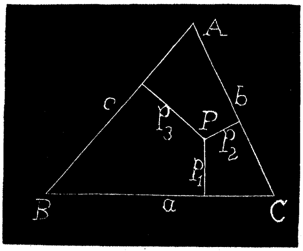

Fig. 1.

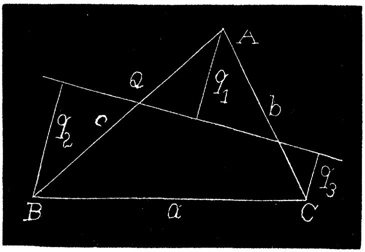

Fig. 1′.

In figure

1, p_1, p_2, p_3 are the perpendicular distances of a point P, from the sides of the coördinate triangle.

1′, q_1, q_2, q_3 are the perpendicular distances of a line Q, from the vertices of the coördinate triangle.

The three

point coördinates of P are expressed as follows :

$$\begin{aligned}\rho x_1 &= p_1 \varkappa_1\\ \rho x_2 &= p_2 \varkappa_2\\ \rho x_3 &= p_3 \varkappa_3\end{aligned} \qquad (1)$$

line coördinates of Q are expressed as follows :

$$\begin{aligned}\sigma u_1 &= q_1 \lambda_1\\ \sigma u_2 &= q_2 \lambda_2\\ \sigma u_3 &= q_3 \lambda_3\end{aligned} \qquad (1)'$$

The $\varkappa$'s and λ's are six constants which might be chosen at pleasure, but for convenience are chosen in a particular way.

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*For a fuller treatment see Clebsch, Vorlesungen ueber Geometrie, S. 27-29, S. 62-78.